Zero-bandwidth mode in a split-ring-resonator-loaded one-dimensional photonic crystal

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We demonstrate that the resonance of metallic split-ring resonators can interact with Bragg phase interference effects in a surrounding one-dimensional photonic crystal in such a way that a zero-bandwidth mode arises within the photonic band gap of the crystal. The band can also be designed to exhibit forward or backward slow-light propagation. We present a very simple model to explain these phenomena and verify the results with numerical simulations. The dynamic tuning of the structure has potential for stopping light or time-reversal applications.

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Great advances in the control of electromagnetic fields by periodic structures have been made in recent years. On one side, metamaterials exhibit surprising material properties such as left-handed propagation,[1] typically making use of split-ring resonators (SRRs) to achieve a negative permeability.[2] On the other side, photonic crystals (PhCs) make use of interference between incident and reflected waves to achieve exciting phenomena such as photonic band gaps. Both structures can be used for slow-light propagation, which potentially allows the creation of optical buffers and delay lines, as well as enhanced nonlinear effects.[3] In this Brief Report, we combine both structures in order to achieve slow-light propagation but we make use of phase interference effects between resonators. It is known that interference between excitation pathways lies behind the slow-light phenomena observed in electromagnetically induced transparency (EIT) (Ref. 4) in atomic media: the strong dispersion arising due to quantum interference[5] yields a very slow group velocity around the transparency window. Classical analogs of EIT have been identified[6] and coupled-resonator-induced transparency (CRIT) has been proposed[7] in which two nearby coupled optical resonators exhibit phase interference effects—analogous to EIT—resulting in slow-light propagation. All those EIT/CRIT interference effects can be called *direct* since the interfering entities share the same spatial location and their periodicity does not play a significant role. In contrast, *indirect* effects exist on resonators which are indirectly coupled via the propagating modes between them. This is the case in systems of periodically spaced optical resonators, named resonant photonic band-gap structures (RPBGs) (Ref. 8) or indirect coupled resonator optical waveguides (CROWs),[9] on which the high dispersion around resonance due to phase interference effects yields a very low group velocity of light—resembling EIT and CRIT—under the condition that the resonance frequency of the resonators matches a Bragg frequency of the optical lattice. Analogous structures have also been analyzed using interface response theory.[10]

A fundamental limit of *static* slow-light devices is the delay-bandwidth product[11] which limits the available bandwidth of the pulses that can be delayed. This limit can be overcome—with certain limitations[12]—by using *dynamic* structures. The dynamic reduction in the group velocity down to zero, for light storage overcoming the delay-bandwidth product limitation, has been performed in atomic EIT schemes[13] as well as proposed in classical EIT analogs[14,15] and also in periodically spaced resonators (RPBGs/indirect CROWs) implemented with Bragg-spaced quantum-well resonators.[16–18]

In this Brief Report, we combine the worlds of PhCs and subwavelength metamaterials to implement a periodic structure exhibiting *indirect* interference effects which can result in slow light. In particular, we analyze the electromagnetic field propagation through an infinite SRR-loaded PhC, whose unit cell is shown in Fig. 1. We only consider one-dimensional propagation with the polarization as indicated in Fig. 1. We proceed in the same way as Syms *et al.*:[19] the SRR is modeled as a simple LC circuit, coupled by a mutual inductance M to a transmission line element representing the surrounding medium. We model the PhC by including *finite-length* transmission lines that represent the two alternating PhC layers, through which incident and reflected waves will propagate and will indirectly couple the SRRs between them; such model is a simple means to account for Bragg effects. These effects will lead to very interesting slow-light phenomena as will be shown below. Figure 2(a) shows the proposed model for the unit cell. Its transmission matrix $\mathbf{T}$ relates the output fields E_x and H_y with those at the input, and can be written as

$$\mathbf{T} = \mathbf{TL}(\varepsilon_2,\mu_2,a_2)\cdot\mathbf{TL}[\varepsilon_1,\mu_1,a_1(1-p)] \cdot\mathbf{SRR}(L,C,M)\cdot\mathbf{TL}(\varepsilon_1,\mu_1,a_1 p), \qquad (1)$$

where $\mathbf{TL}(\varepsilon_i,\mu_i,a_i)$ represents the transmission matrix of a transmission line with propagation constant $\beta_i=\omega\sqrt{\mu_i\varepsilon_i}$, characteristic impedance $Z_i=\sqrt{\mu_i/\varepsilon_i}$, and length a_i, p describes the relative position of the SRR in the PhC medium, and $\mathbf{SRR}(L,C,M)$ represents the transmission matrix of a SRR.[1,19] Equating $2\cos(ka)$ with the sum of the diagonal elements of $\mathbf{T}$ yields the dispersion relation of the periodic structure,[20] given by

$$2\cos(ka) = f(\omega) = f_{\mathrm{PhC}}(\omega) + \frac{\omega_0 M^2}{L}\left(\frac{\omega/\omega_0}{1-\omega_0^2/\omega^2}\right)C(\omega), \qquad (2)$$

where k is the Bloch wave vector, $a=a_1+a_2$ is the periodicity of the unit cell, $\omega_0=1/\sqrt{LC}$ is the resonance frequency of the

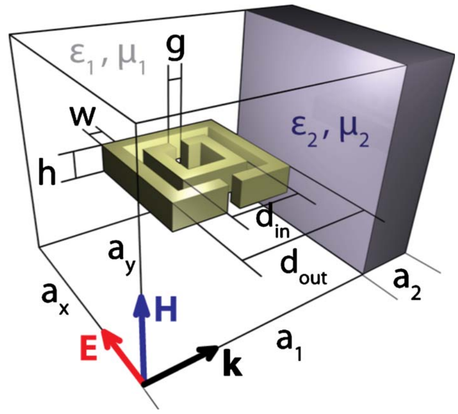

FIG. 1. (Color online) Unit cell of the SRR-loaded PhC.

SRRs, and the terms $f_{\mathrm{PhC}}(\omega)$ and $C(\omega)$ (Ref. 21) do not depend on the SRR response. This model could be also applied to other resonant particles such as high-index dielectric disks or rings, whose circuit model is equivalent to that of the SRRs.

Figures 2(b)–2(d) show the dispersion relation described by the model (2) for three different values of ω_0 with respect to the lower ω_1 and upper ω_2 edges of the PhC band gap. When $\omega_0<\omega_1$ [Fig. 2(b)], we are in the long-wavelength regime and the dispersion relation is very similar to that of a metamaterial SRR medium. When $\omega_0>\omega_2$ [Fig. 2(d)], the bands are similar to the previous case but folded back into the first Brillouin zone. The most interesting case occurs when $\omega_1<\omega_0<\omega_2$ [Figs. 2(c), 3(a), and 3(b)], where the

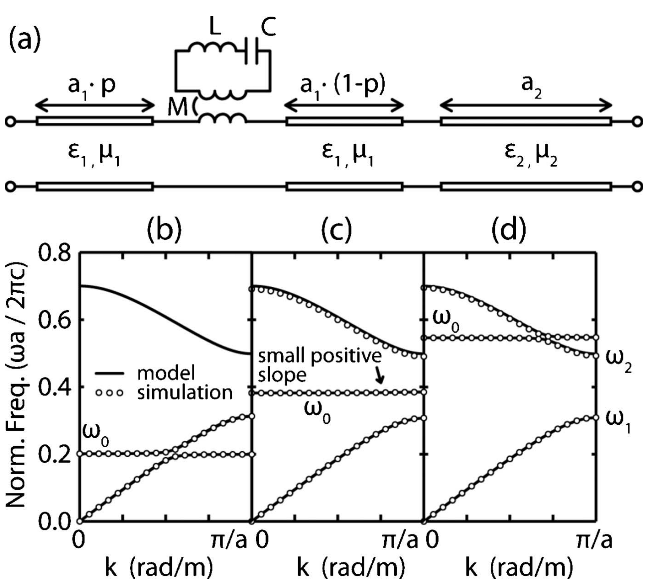

FIG. 2. (a) Proposed circuit model for the SRR-loaded PhC unit cell. (b)–(d) Modeled (solid lines) and simulated (circles) normalized dispersion relations for (b) $\omega_0<\omega_1$, (c) $\omega_1<\omega_0<\omega_2$, and (d) $\omega_0>\omega_2$. The model parameters are $p=0.5$, (b) $\omega_0 a/2\pi c=0.2$, (c) $\omega_0 a/2\pi c=0.381$, (d) $\omega_0 a/2\pi c=0.5463$, and $\omega_0 M^2/L=5$ in all cases. The PhC has $a_1=70a/75$ of air and $a_2=5a/75$ of dielectric $n_2=3.45$. The simulated SRR structure has $d_{out}=22s$, $d_{in}=18s$, $g=w=1s$, $h=9s$, and $a_x=a_y=100s$, where s is a reference feature size (b) $s=a/75$, (c) $s=0.5a/75$, and (d) $s=0.35a/75$.

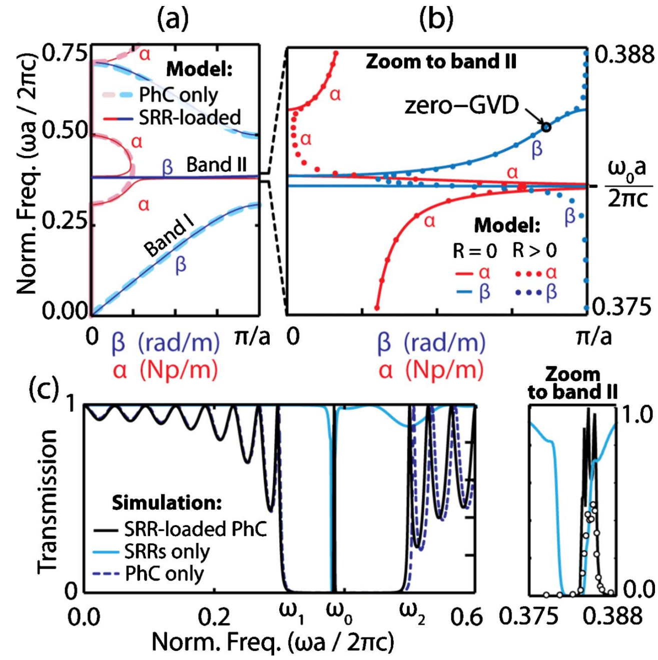

FIG. 3. (Color online) (a) Complex dispersion relation of the SRR-loaded PhC (thin solid line) and the PhC alone (thick dashed line) using same parameters as Fig. 2(c). (b) Zoom to Band II for the SRR-loaded PhC for lossless SRRs (solid lines) and lossy SRRs with $R/L=1\times10^{7}\ \mathrm{s}^{-1}$ (dotted lines), a value chosen arbitrarily within the expected range. (c) Left: simulated power transmission spectrum through eight unit cells of a lossless SRR-loaded PhC, SRRs only, and PhC only. Right: a zoom around ω_0. Dots in zoomed view correspond to simulation with realistic copper metal $\rho_{\mathrm{Cu}}=5.8\times10^{7}\ \Omega^{-1}\ \mathrm{m}^{-1}$ and $a=75$ mm.

SRR resonance occurs within the photonic band gap. This resembles the setup of SRRs introduced in a band gap created by a wire medium,[22] a waveguide below cutoff[23] or an etched microstrip ground plane.[24] However, in the present case, we cannot apply an effective-medium interpretation. We interpret our results by noting that the SRR resonant mode does not coexist with propagating modes in the PhC, and thus, neither coupling nor anticrossing take place, resulting in a flat dispersion relation with very small group velocity within the whole Brillouin region. A zoom into the passband, showing the complex Bloch wave vector ($k=\beta-j\alpha$), is shown in Fig. 3(b). The band has an inflection point at which the *group-velocity dispersion* is zero.

To check the model, Figs. 2(b)–2(d) include dispersion-relation simulations using the eigenmode solver of CST MICROWAVE STUDIO. Figure 3(c) shows a simulated transmission spectrum. Simulations show an almost exact agreement with the model. A source of disagreement is the slight redshift of the PhC bands when introducing the SRRs in simulations, specially the upper band, which can be interpreted as if the presence of the SRRs causes the PhC to "see" an effective medium with index n_{1eff} instead of n_1. This redshift becomes noticeable if the filling fraction of the SRRs is increased (larger SRR size or reduced transversal periodicity). Losses are an important practical issue, Fig. 3(b) shows, using dots, the complex dispersion relation when a resistance $R>0$ in series with L and C is used to account for metal

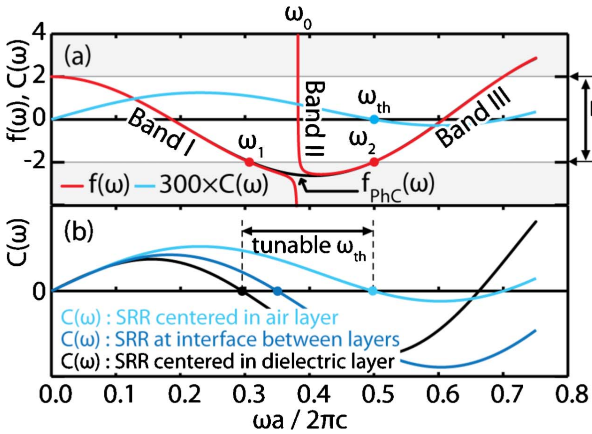

FIG. 4. (Color online) (a) Plot of $f(\omega)$, $f_{\mathrm{PhC}}(\omega)$, and $C(\omega)$ for the same parameters used in Fig. 2(c). (b) Plot of $C(\omega)$ for the same parameters varying the position p of the SRRs.

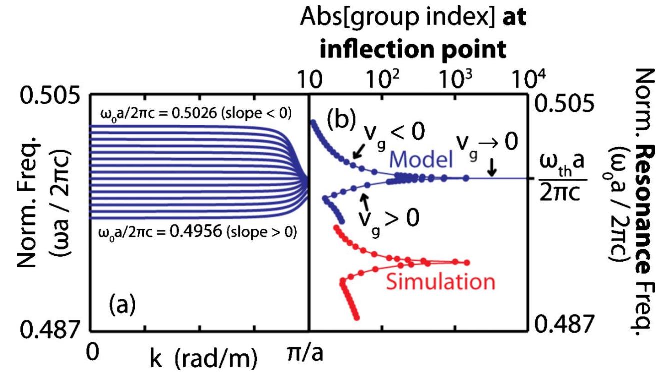

FIG. 5. (Color online) (a) Dispersion relation of Band II of the SRR-loaded PhC model for different values of resonant frequencies ω_0 varied around a fixed ω_{th}. (b) Modeled (blue, darker gray) and simulated (red, lighter gray) absolute value of the group index of Band II at its inflection point as a function of ω_0.

absorption in the SRR, and Fig. 3(c) shows, using dots, a realistic transmission simulation for an arbitrarily chosen size (a=75 mm resulting in SRR resonance around 1.5 GHz), using a realistic copper metal for the SRRs. The results indicate that an experimental demonstration at microwave or terahertz frequencies is at hand.

Much more interesting phenomena related to the interaction of the SRRs with the PhC periodicity can be deduced by taking a closer look at the model. From Eq. (2), we see that a propagating band exists at each frequency ω only if $-2 < f(\omega) < 2$. The first term of $f(\omega)$ is $f_{\mathrm{PhC}}(\omega)$, which constitutes the dispersion relation of the PhC without SRRs, and defines the PhC band gaps whenever $|f_{\mathrm{PhC}}(\omega)| > 2$. The second term in $f(\omega)$ is due to the SRRs, and it is an asymptotic function which diverges at ω_0. The total function $f(\omega)$ for the case $\omega_1 < \omega_0 < \omega_2$ is plotted in red (dark gray) in Fig. 4(a), where the three propagating bands previously shown in Figs. 2(c) and 3 are clearly observed. The width of the asymptote (and consequently the bandwidth of Band II) is determined by the product of the terms $\omega_0 M^2/L$ and $C(\omega)$. The sign of the slope of the two branches of the asymptotic function (and consequently the sign of the slope of Band II) are determined directly by the sign of $C(\omega_0)$. Let us call threshold frequency ω_{th} to the first frequency $\omega > 0$ at which $C(\omega)$ changes sign [cyan (light gray) in Fig. 4(a)]. It follows that the slope and bandwidth of Band II is determined by the relative position of ω_0 with respect to ω_{th}. Therefore, Band II can be forward ($\omega_0 < \omega_{th}$), backward, ($\omega_0 > \omega_{th}$) or completely flat with zero bandwidth ($\omega_0 \to \omega_{th}$). Such result resembles EIT (and its analogs) as expected since it arises from interference effects between resonators. In this case the form of $C(\omega)$ (Ref. 21) suggests that the phase interference is *indirect* (i.e., through propagating modes), in analogy to other indirect-CROW structures.[8–10] The effect can be interpreted as destructive field interference, taking place between the SRRs and the impinging propagating modes in the medium, that cancels out the total power flux. Unlike *direct* interference slow-light schemes such as EIT, in this case only one resonator (plus periodicity) is required per unit cell. The phenomenon is illustrated in Fig. 5 in which the dispersion relation of Band II as well as its group index at the inflection point for varying values of ω_0 around ω_{th} are shown. Simulations show the expected slight redshift of ω_{th} due to the previously mentioned change in n_1 into n_{1eff} [which affects $C(\omega)$ and thus ω_{th}].

We would like to highlight the difference between directly coupled resonator structures [standard CROWs (Ref. 25)] and the case presented here (indirect CROW) in which resonators are coupled through mode propagation in the medium, showing phase interference effects similar to those in EIT, thus allowing a decrease in group velocity (and consequently the bandwidth) down to zero. The comparison is analogous to that made by Khurgin[26] between standard CROWs and EIT media. Notice that magnetoinductive coupling between SRRs is not included in our model and does not seem to play a significant role.

An adiabatic dynamic tuning of the slope of Band II [for example, by modulating the index of the dielectric n_2, which in turn modifies $C(\omega)$ and thus ω_{th}] can lead to many sophisticated capabilities: the scheme satisfies the criteria identified by Yanik and Fan[27] for the stopping and storage of light pulses with no delay-bandwidth limitation[14] as well as the time reversal of electromagnetic pulses.[15] Figure 4(b) shows how the term $C(\omega)$ and thus ω_{th} can be tuned by varying the position of the SRRs within the unit cell (parameter p in the model). As seen, this allows us to place ω_{th} anywhere within the photonic band gap. A value of $p \approx 0.2$ with the parameters used in Fig. 2 places ω_{th} in the center of the band gap. If the tunable band exists in the center of a PhC band gap, a relatively *fast* adiabatic modulation of the refractive index (which is a critical practical consideration[12]) can be performed with negligible scattering into the neighboring bands.[14] In the particular case in which a_2=0, we have $C(\omega) = \sin(\beta_1 a_1)/Z$, and the resulting structure is an example of RPBGs/indirect CROWs as studied in Refs. 8 and 9, showing an extremely slow group-velocity band when the resonators are periodically spaced by a multiple of half wavelength. That phenomenon has been used in Bragg-spaced quantum-well resonators[16,17] to achieve slow-light applications, and the practical limitations have been thoroughly discussed,[17] as well as the combination of slow light with time-reversal capabilities to achieve distortion compensation in a reflection scheme.[18] All those considerations also apply to our proposed structure. Some advantages of our proposed metamaterial-inspired structure are the freedom in the design of the SRRs and in the number of achievable unit cells. In addition, we introduce a PhC (dielectric n_2). This increases the complexity of the phase interference phenomena between SRRs, allowing an increase in the bandwidth of the band gap around the slow-light band, and enables the tuning of ω_{th} as previously discussed. Such apparent complexity arises simply from the in-line reflections produced by the PhC interfaces. In Ref. 28, the interaction between a cavity resonance and in-line reflections produced at both sides of the cavity is shown to result in complex interference phenomena that give rise to sharp and asymmetric Fano line shapes in the response. Our structure can be regarded as the simple periodization of such system. Fano line shapes are therefore characteristic of SRR-loaded PhCs, showing a strong transmission dip at the resonance frequency always adjacent to the slow-light band.

In conclusion, we have modeled a structure exhibiting *indirect* phase interference effects between SRRs placed inside a PhC which result in a slow-light transmission band, confirmed with numerical simulations, which can ultimately show zero bandwidth. The result is similar to other slow-light structures based on *direct* (EIT schemes) and *indirect* (RPBGs/indirect CROWs) interference. The zero bandwidth occurs when the resonance frequency ω_0 of the resonators approaches the so-called threshold frequency ω_{th}, determined by the parameters of the PhC and the position of the resonators within the unit cell. The detuning of ω_0 with respect to ω_{th} determines whether the band has positive, negative, or zero slope. An adiabatic tuning of such structure has potential for nontrivial capabilities such as stopping light with no delay-bandwidth product limitation and time-reversal operations.

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[1] L. Solymar and E. Shamonina, *Waves in Metamaterials* (Oxford University Press, New York, 2009).

[2] J. B. Pendry, A. J. Holden, D. J. Robbins, and W. J. Stewart, IEEE Trans. Microwave Theory Tech. **47**, 2075 (1999).

[3] T. Baba, Nat. Photonics **2**, 465 (2008).

[4] S. E. Harris, Phys. Today **50**, No. 7, 36 (1997).

[5] Y. Q. Li and M. Xiao, Phys. Rev. A **51**, 4959 (1995).

[6] A. G. Litvak and M. D. Tokman, Phys. Rev. Lett. **88**, 095003 (2002).

[7] D. D. Smith, H. Chang, K. A. Fuller, A. T. Rosenberger, and R. W. Boyd, Phys. Rev. A **69**, 063804 (2004).

[8] N. H. Kwong, J. Sipe, R. Binder, Z. Yang, and A. Smirl, in *Slow Light: Science and Applications*, edited by J. B. Khurgin and R. S. Tucker (CRC Press, Cleveland, 2009).

[9] Y. Xu, Y. Li, R. K. Lee, and A. Yariv, Phys. Rev. E **62**, 7389 (2000).

[10] J. O. Vasseur, A. Akjouj, L. Dobrzynski, B. Djafari-Rouhani, and E. H. El Boudouti, Surf. Sci. Rep. **54**, 1 (2004).

[11] G. Lenz, B. J. Eggleton, C. K. Madsen, and R. E. Slusher, IEEE J. Quantum Electron. **37**, 525 (2001).

[12] J. B. Khurgin, Opt. Lett. **30**, 2778 (2005).

[13] D. F. Phillips, A. Fleischhauer, A. Mair, R. L. Walsworth, and M. D. Lukin, Phys. Rev. Lett. **86**, 783 (2001).

[14] M. F. Yanik, W. Suh, Z. Wang, and S. Fan, Phys. Rev. Lett. **93**, 233903 (2004).

[15] M. F. Yanik and S. Fan, Phys. Rev. Lett. **93**, 173903 (2004).

[16] M. Hübner, J. Kuhl, T. Stroucken, A. Knorr, S. W. Koch, R. Hey, and K. Ploog, Phys. Rev. Lett. **76**, 4199 (1996).

[17] Z. S. Yang, N. H. Kwong, R. Binder, and A. L. Smirl, J. Opt. Soc. Am. B **22**, 2144 (2005).

[18] Z. S. Yang, N. H. Kwong, R. Binder, and A. L. Smirl, Opt. Lett. **30**, 2790 (2005).

[19] R. R. A. Syms, E. Shamonina, and V. Kalinin, J. Appl. Phys. **97**, 064909 (2005).

[20] L. Brillouin, *Wave Propagation in Periodic Structures* (McGraw-Hill, London, 1946).

[21] The terms are given by $f_{\mathrm{PhC}}(\omega)=2\cos(\beta_1 a_1+\beta_2 a_2)+[2-(Z_1/Z_2)-(Z_2/Z_1)]\sin(\beta_1 a_1)\sin(\beta_2 a_2)$ and $C(\omega)=\frac{1}{Z_1^2 Z_2}\{Z_1 Z_2 \sin(\beta_1 a_1)\cos(\beta_2 a_2)+1/2\sin(\beta_2 a_2)\{(Z_1^2+Z_2^2)\cos(\beta_1 a_1)+(Z_1^2-Z_2^2)\cos[\beta_1 a_1(1-2p)]\}\}$.

[22] D. R. Smith, W. J. Padilla, D. C. Vier, S. C. Nemat-Nasser, and S. Schultz, Phys. Rev. Lett. **84**, 4184 (2000).

[23] R. Marqués, J. Martel, F. Mesa, and F. Medina, Phys. Rev. Lett. **89**, 183901 (2002).

[24] M. Navarro-Cía, F. Falcone, M. Beruete, I. Arnedo, J. Illescas, A. Marcotegui, M. A. G. Laso, and T. Lopetegui, Microwave Opt. Technol. Lett. **49**, 2689 (2007).

[25] A. Yariv, Y. Xu, R. K. Lee, and A. Scherer, Opt. Lett. **24**, 711 (1999).

[26] J. B. Khurgin, J. Opt. Soc. Am. B **22**, 1062 (2005).

[27] M. F. Yanik and S. Fan, Phys. Rev. A **71**, 013803 (2005).

[28] S. Fan, Appl. Phys. Lett. **80**, 908 (2002).